SCATTERING BY PERIODIC METAL SURFACES WITH SINUSOIDAL HEIGHT PROFILE - A NEW THEORETICAL APPROACH

G. Whitman
New Jersey Institute of Technology
Newark, New Jersey

F. Schwering
U.S. Army Electronics Command
Fort Monmouth, New Jersey

Abstract

A new theory of scattering by periodic metal surfaces is presented. The approach reduces the scatter problem to solving a linear system whose coefficients are obtained in closed form. The theory is thus amenable to efficient computer evaluation. Numerical results have shown that for depth of grooves less than a wavelength and for unrestricted groove widths reliable and comparable, if not more accurate, data is obtained at minimal computational cost.

Introduction

Many of the procedures used to study scattering from periodic surfaces are based on the original approach used by Lord Rayleigh in 1878,[1] in which a discrete spectrum of outgoing plane waves is assumed for the scattered field. Difficulties occur in applying boundary conditions since this form of the scattered field does not in general apply at the scatter surface.[2,3] Only recently, with the availability of high speed computers, has the evaluation of rigorous integral representations of solutions to these problems become practical and reliable results been obtained.[4-8] In this study, a new theoretical approach to the problem of scattering by periodically corrugated metal surfaces is presented. The induced current distribution on the perfectly conducting surface is assumed to be a Fourier series (whose fundamental spatial period is equal to the width of the surface grooves - Floquet's theorem) multiplied by the physical optics current density or a suitably chosen modification of it. The unknown coefficients of the Fourier series are determined from the condition that the total field below the scatterer be zero. This method converts the solution of the scatter problem, for many periodic configuration, to the solution of a linear system whose matrix coefficients reduce to closed form expressions in terms of well-known functions. The unknown Fourier coefficients become amenable to efficient computer evaluation after appropriate truncation of the linear system. With the known induced current distribution, closed form expressions are then obtained for the scattered field above the metal surface. By increasing the order of the system evaluated, it should in principle be possible to improve the accuracy of the approach without limit (though this has not yet been proven).

Experimentally, it has been established that TM-polarization is superior to TE-polarization in minimizing false guidance of microwave scanning beam landing systems by substantially reducing specular reflection from large (periodic) metal structures near runways.[9] TE(TM)-polarization is characterized by an electric (magnetic) field directed parallel to the surface grooves. Numerical results confirm this experimental evidence, particularly in the practical range of low incidence angles (near grazing).

Conservation of power and reciprocity are used to numerically test the accuracy of this new theory. Its dependence on surface groove depth and width is discussed. Finally, comparisons are made of data obtained by this method with results based on the numerical schemes of Tong and Senior,[6] and of Zaki and Neureuther.[4,5]

Scatter Problem and Solution

A uniform plane wave (with suppressed time dependence $\exp(i\omega t)$ is incident at an angle θ upon a metal sinusoidally varying surface defined by $z_o = h\sin(2\pi x_o/d)$, $-\infty \le x_o, y \le \infty$ (see Fig. 1).

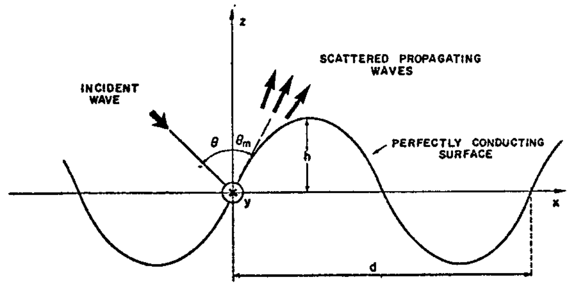

Fig. 1. Sinusoidal metal surface illuminated by plane wave; coordinates and geometry of scatter problem.

Due to the periodic nature of the surface, the scatter field in the region $z>z_o$ can be represented as a discrete spectrum of propagating and evanescent plane waves (space harmonics):

$$U^q = \sum_{m=-\infty}^{\infty} U_m^q \, e^{-ik(x\sin\theta_n + z\cos\theta_n)}, \quad U^q = E', H'' . \qquad (1)$$

$E'(H'')$ and $E_m'(H_m'')$, respectively, represent the scattered y-directed electric (magnetic)field and associated space harmonic amplitudes resulting from a TE(TM)-polarized incident plane wave $E_i'(H_i'')$. Wavenumber and wavelength are related by $k=2\pi/\lambda$ and scatter angles θ_m are given by $\sin\theta_m = \sin\theta + m\lambda/d$.

Complex amplitudes U_m^q can be expressed in terms of integrals whose kernels involve the unknown induced surface current distribution K^q. These integrals reduce to closed form expressions on assuming

$$K^q = K_p^q F^q, \quad F^q = \sum_{n=-\infty}^{\infty} C_n^q \, e^{-i2\pi n x_o/d}, \qquad (2)$$

where for TE-polarization

$$K_p' = 2\left(\frac{\varepsilon}{\mu}\right)^{\frac{1}{2}} \frac{\cos\theta}{[1+(dz_o/dx_o)^2]^{\frac{1}{2}}} \, e^{-ik(x_o\sin\theta - z_o\cos\theta)}$$

and for TM-polarization

$$K_p'' = -2\, e^{-ik(x_o\sin\theta - z_o\cos\theta)} .$$

K_p' is a modified form and K_p'' the actual form of the physical optics approximation of the current densities.* F^q is a Fourier series whose fundamental period equals the spatial period of the metal scatterer. Space harmonic amplitudes then can be shown to take the form

$$E'_m = \frac{\cos\theta}{\cos\theta_m}\sum_{m=-\infty}^{\infty}(-1)^{m+n+1}J_{m+n}(\alpha_{1m})C'_n \qquad (3a)$$

for TE-polarization and

$$H''_m = \sum_{m=-\infty}^{\infty}(-1)^{m+n+1}\left[1+(m+n)\frac{p_m}{\alpha_{1m}}\right]J_{m+n}(\alpha_{1m})C''_n \qquad (3b)$$

for TM-polarization. J_{m+n} are Bessel functions of order $m+n$ and

$$\alpha_{1m} = 2\pi(h/\lambda)[\cos\theta + \cos\theta_m],\quad p_m = 2\pi(h/\lambda)\tan\theta_m. \qquad (3c)$$

The Fourier coefficients C^q_n in (3) are found by imagining the metal scatterer to be replaced by K^q. This current must then radiate a field into the lower half-space ($z<z_o$). The incident field is allowed to penetrate this region so that a net zero field results. This means that the zero order space harmonic cancels the incident wave (they travel in the same direction) and that all higher order space harmonics are zero, i.e.,

$$U^q_o + U^q_i = 0 \quad \text{for } m=0, \quad U^q_m = 0 \text{ for } m\neq 0 . \qquad (4)$$

Using equations (2), (4) and integral expressions for the field radiated by K^q into region $z<z_o$, a linear system of equations for the Fourier coefficients is obtained. This linear system is amenable to efficient computer evaluation; its matrix elements are Bessel functions of complex arguments.

Numerical Results, Accuracy Checks, Program Limitations

A computer program for numerical evaluation of the linear system under discussion has been constructed. Program outputs have included the induced surface current densities, the amplitudes and phases of the scattered space harmonics and accuracy checks based on conservation of power and reciprocity. Numerical data was acquired for surface profiles (1) $d/\lambda = 2.5$, $h/\lambda = 0.375$ and (2) $d/\lambda = 1.3$, $h/\lambda = 0.1333$, which approximately describe surfaces of practical interest.

Figs. 2 and 3 show the power P_m of the (propagating) space harmonics of surface profile 2 plotted vs. θ for TE- and TM-polarization of the incident plane wave, respectively. P_m denotes the power transmitted by the m^{th} space harmonic through unit area of planes z = constant assuming the incident power per unit area is unity. Observe that in the range of large incidence angles ($\theta>60^o$) the specular reflection coefficient (P_o) is significantly greater for TE- than for TM-polarization. The curves show pronounced Rayleigh-Wood type anomalies indicated on the figures by small arrows. As expected, S-type anomalies (TM case) are stronger than P-type anomalies (TE case).

Conservation of power and reciprocity are used to check the accuracy of numerical values. The former requires that

$$\sum_m P_m = 1, \quad P_m = |U^q_m|^2\cos\theta_m/\cos\theta, \qquad (5)$$

with the summation extending over all propagating spectral orders. Reciprocity stipulates that

$$(U^q_m)^a\cos\theta^a_m = (U^q_m)^b\cos\theta^b_m , \qquad (6)$$

where unity amplitude waves incident from directions θ^a and θ^b produce space harmonics of order m scattered in directions $\theta^{a,b}_m = -\theta^{b,a}$.

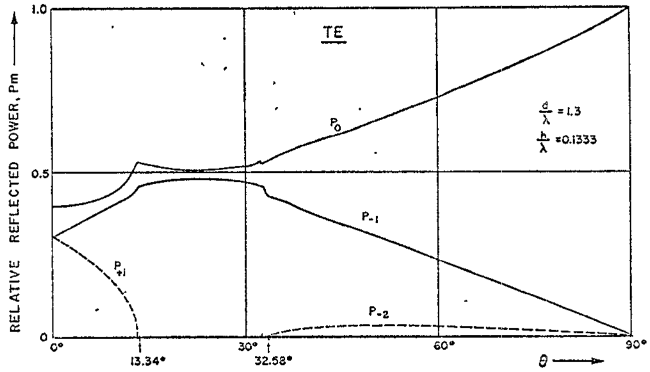

Fig. 2. Powers P_m of propagating space harmonics vs. incidence angle θ. Spectral orders: m = +1, 0,-1,-2.

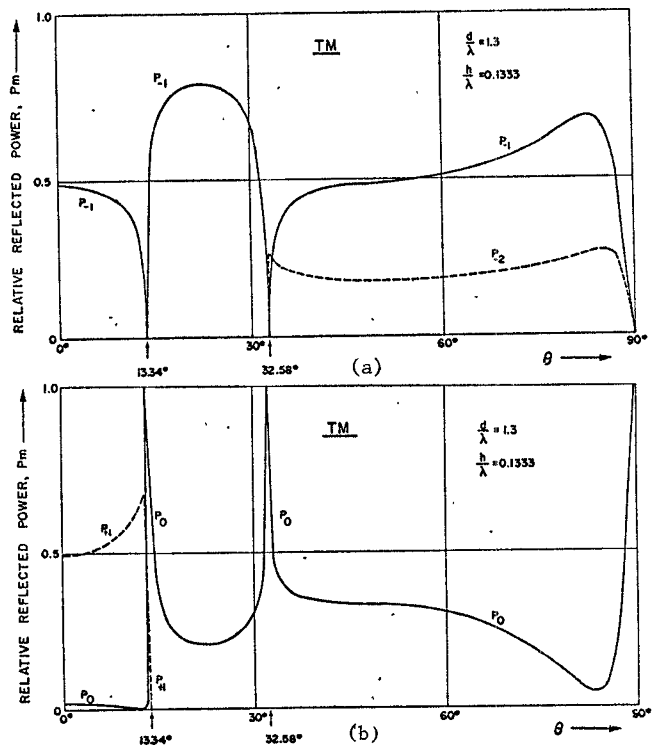

Fig. 3. Powers P_m of propagating space harmonics vs. incidence angle θ. Spectral orders: (a) m = -1,-2; (b) m = 0,+1.

In Table 1, the relative power errors, $\varepsilon = (\sum_m P_m - 1)\cdot 100$, are listed as functions of θ for profile 1. The table shows that the relative power errors are generally in the order of 1% or less. Accuracy was

*Integral equations, obtained on use of the boundary condition $E_{tan}=0$ at the metal surface, yield when evaluated in the vicinity of the singularities of their kernels these expressions for K^q_p.

found to fall for incidence angles near Rayleigh-Wood type anomalies. For surface profile 2, accuracy was very high with $|\varepsilon| \sim 10^{-4}\%$.

θ^o	0	10	20	30	40	50	60	70	80	85
$\varepsilon'(\%)$	0.05	-0.12	0.19	0.24	-1.16	0.41	0.58	0.57	0.17	0.06
$\varepsilon''(\%)$	0.44	-1.84	-0.31	0.02	-6.32	-2.66	-0.83	0.04	0.17	0.38

Table 1. Accuracy check (power criterion) on numerical results for $d/\lambda=2.5$, $h/\lambda=0.375$ ε' and ε'' corresponds to TE- and TM-polarizations, respectively.

In Table 2, the results of a reciprocity check are shown for profile 1. In the TE case, relative amplitude errors are generally below 0.5% - larger errors are due to smaller amplitudes - and phase errors remain below 1%. In the TM case, discrepancies are appreciably larger.

$\theta^a=-\theta^b_m$	$\theta^b=-\theta^a_m$	m	Error(%)	$\phi^a_m - \phi^b_m$
A. TE-Polarization				
60	47.22	-4	0.08	0.11
60	19.51	-3	-0.15	-0.03
60	-3.79	-2	-0.32	0.00
60	-27.78	-1	0.04	-0.64
60	-60.00	0	0.00	0.00
B. TM-Polarization				
60	47.22	-4	-0.68	-0.01
60	19.51	-3	0.13	-0.42
60	-3.79	-2	0.47	-0.07
60	-27.78	-1	0.85	0.94
60	-60.00	0	0.00	0.00

$$\text{Error}(\%) = \frac{|U^q_m|^a \cos\theta^a_m - |U^q_m|^b \cos\theta^b}{|U^q_m|^b \cos\theta^b_m} \times 100$$

$\phi^{a,b}_m = \arg(U^q_m)^{a,b}$; all angles in degrees.

Table 2. Accuracy check (reciprocity criterion) on complex amplitudes of space harmonics for $d/\lambda=2.5$, $h/\lambda=0.375$.

(a)

d/λ	h/λ	θ^o	WS $\|\varepsilon'\|$	WS $\|\varepsilon''\|$	TS $\|\varepsilon'\|$	TS $\|\varepsilon''\|$
0.2	0.1	60	0.02	0.12	0.26	1.50
0.4	0.2	0	1.64	2.26	0.76	1.77
0.4	0.2	60	0.10	0.62	1.36	0.84
1.9	0.25	0	0.0	0.04	0.04	0.26

(b)

d/λ	h/λ	θ^o	WS $\|\varepsilon'\|$	WS $\|\varepsilon''\|$	ZN $\|\varepsilon'\|$	ZN $\|\varepsilon''\|$
1.5	0.1	60	$0.44\cdot10^{-4}$	$0.46\cdot10^{-4}$	$0.69\cdot10^{-2}$	1.18
2.5			$0.44\cdot10^{-4}$	$0.21\cdot10^{-4}$	$0.15\cdot10^{-3}$	0.78
3.5			$0.40\cdot10^{-4}$	$0.29\cdot10^{-5}$	$0.33\cdot10^{-3}$	1.31
1.5	0.25	0	$1.18\cdot10^{-2}$	0.12	$1.77\cdot10^{-2}$	0.60
2.5			$0.30\cdot10^{-4}$	$1.42\cdot10^{-5}$	$0.26\cdot10^{-2}$	$0.93\cdot10^{-1}$
3.5			$0.85\cdot10^{-6}$	$1.62\cdot10^{-6}$	$0.41\cdot10^{-2}$	$0.42\cdot10^{-1}$
1.5	0.25	60	$0.82\cdot10^{-3}$	$0.55\cdot10^{-2}$	$1.72\cdot10^{-2}$	0.23
2.5			0.23	0.27	$0.16\cdot10^{-2}$	0.57
3.5			3.07	4.14	$0.34\cdot10^{-2}$	1.16

Table 3. Comparison of relative power errors: Whitman-Schwering vs. (a) Tong-Senior and (b) Zaki-Neureuther.

Comparison of the numerical accuracy of this new method (WS) with that of Tong and Senior (TS) and of Zaki and Neureuther (ZN) are given in Tables 3a and 3b, respectively. Table 3a was compiled from tables in reference 6 and Table 3b from the use of the ZN-computer program. For the d/λ values tested, the WS-program was found to run on the average $\sim$10 times faster than the ZN-program.

The accuracy of the computed results has been found to be critically dependent on the depth of the surface grooves, i.e., on $2h/\lambda$. The power and reciprocity criterion are excellently satisfied at small $2h/\lambda<0.2$ and well satisfied up to $2h/\lambda \sim 1.0$. No limitations have been encountered with regard to d/λ values.

Conclusion

A new rigorous approach to the problem of plane wave scattering from periodic metal surfaces was presented. Numerical evaluation verified that the TE-polarized specular (power) reflection coefficient is significantly larger than the associated TM one, particularly when $\theta>60^o$. Conservation of power and reciprocity were quite adequately satisfied for $h/\lambda \lesssim 0.5$. Numerical results were found to be in excellent agreement with values computed by other methods. The method has proven to be computationally efficient in the range of groove depth smaller than one wavelength.

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